Direct Access for Conjunctive Queries with Negations

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Séminaire KRDB

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Direct Access on Join Queries

Join Queries

Join Query : $Q(x_1, \ldots, x_n) = \bigwedge_{i=1}^{k} R_i(\mathbf{x_i})$

where $\mathbf{x_i}$ is a tuple over $X = \{x_1, \ldots, x_n\}$

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Example:

$$Q(city, country, name, id) = People(id, name, city) \wedge Capitals(city, country)$$

People

id	name	city
1	Alice	Paris
2	Bob	Lens
3	Chiara	Rome
4	Djibril	Berlin
5	Émile	Dortmund
6	Francesca	Rome

Capitals

city	country
Berlin	Germany
Paris	France
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$Q(\mathbb{D})$

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Direct Access

Quickly access $Q(\mathbb{D})[k]$, **the k^{th} element of $Q(\mathbb{D})$**.

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$$Q(\mathbb{D})[2]?$$

$$(Rome, Italy, Chiara, 3).$$

Naive Direct Access

Naive algorithm: materialize $Q(\mathbb{D})$ in an array, sort it. Access.

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In this talk: only lexicographical orders.

Applications

Direct Access generalizes many tasks that have been previously studied:

- **Uniform sampling** without repetitions
- **Ranked enumeration**
- **Counting queries**:
  - how many answers between τ_1 and τ_2?
  - how many answers extend a *partial answer* etc.

Beating the Naive Approach

Beating Naive Direct Access

Naive Direct Access:

- Preprocessing at least $O(\#Q(\mathbb{D}))$.
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Can we have better preprocessing and reasonable access time?

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"Ideal" complexity:

- $O(|\mathbb{D}|)$ preprocessing
- $O(\log|\mathbb{D}|)$ access time

Complexity of Direct Access

Computing $\#Q(\mathbb{D})$ given Q and $\mathbb{D}$ is $\#P$-hard.

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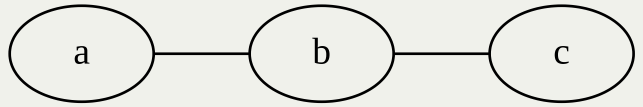
Data complexity assumption: for a fixed Q, what is the best preprocessing $f(|\mathbb{D}|)$ for an access time $O(polylog|\mathbb{D}|)$?

In this work, all presented complexity in data complexity will also be polynomial for combined complexity.

An easy query?

$$Q(a,b,c) = A(a,b) \wedge B(b,c).$$



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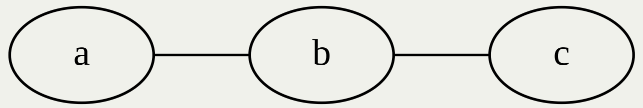

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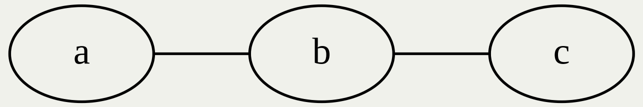

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a	**b**
0	0
1	1
2	1

b	**c**
0	0
0	1
0	2
1	1
1	2

Precomputation :

- $\#Q(0, 0, \_) = 3$
- $\#Q(1, 1, \_) = 2$
- $\#Q(2, 1, \_) = 2$

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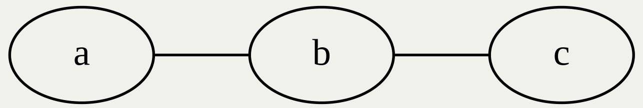

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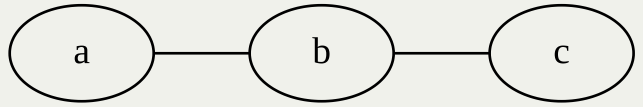
Access $Q[5]$:

- $a = 0, b = 0$: not enough solutions
- $a = 1, b = 1$: enough! 3 solutions smaller than $(1, 1, \_)$
- Look for the **second** solution of $B(1, \_)$: $a = 1, b = 1, c = 2$

A not so easy query

$$Q(a, c, b) = A(a, b) \wedge B(b, c).$$



Direct Access for lexicographical order induced by (a, c, b) **?**

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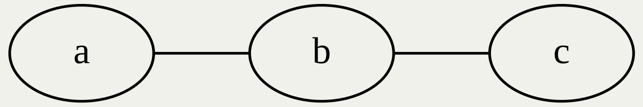

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Reduces to multiplying two $\{0,1\}$-matrices M, N over $\mathbb{N}$:

- $(i,j) \in A$ iff $M[i,j] = 1$, $(j,k) \in N$ iff $N[j,k] = 1$
- $\#Q(i,j,\_) = (MN)[i,j]$
- Direct Access can be used to find $\#Q(i,j,\_)$ with $O(\log|\mathbb{D}|)$ queries.

Characterizing preprocessing time

Given a query Q and order π on its variables, we can compute $\iota(Q, \pi)$ such that:

1. *Tractable Orders for Direct Access to Ranked Answers of Conjunctive Queries*, N. Carmeli, N. Tziavelis, W. Gatterbauer, B. Kimelfeld, M. Riedewald
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- Function ι closely related to fractional hypertree width.

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End of the story?

So, if we understand everything for Direct Access and lexicographical orders, what is **our contribution**?

Signed Join Queries

Definition

$$Q = \bigwedge_{i=1}^{k} P_i(\mathbf{z_i}) \ \bigwedge_{i=1}^{l} \neg N_i(\mathbf{z_i})$$

Negation interpreted **over a given domain** D:

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$\neg N$ on $\{0, 1\}$

x_1	x_2	x_3
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- $\neg N(x_1, ..., x_k)$ encoded with $|D|^k - \#N$ tuples.
- Relation with SAT: $\neg N$ is $x_1 \vee \neg x_2 \vee x_3$

Positive Encoding not Optimal

$Q(x_1, \ldots, x_n) = \neg N(x_1, \ldots, x_n)$, domain $\{0, 1\}$.

Positive encoding: **preprocessing** $O(2^n)$

N

x_1	x_2	x_3
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- $Q(\mathbb{D})[1]$?
- $Q(\mathbb{D})[2]$?
- $Q(\mathbb{D})[3]$?
- $Q(\mathbb{D})[k]$?

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  where $p_k = \{t \in N \mid t \leq k\}$

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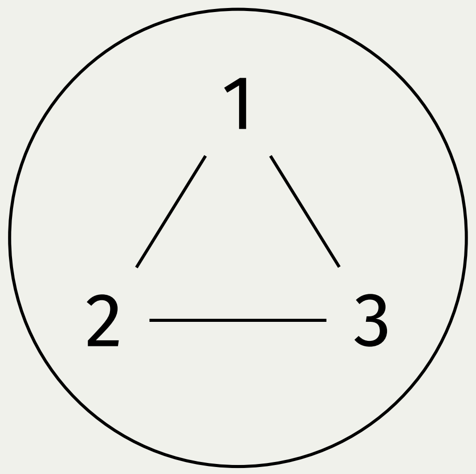
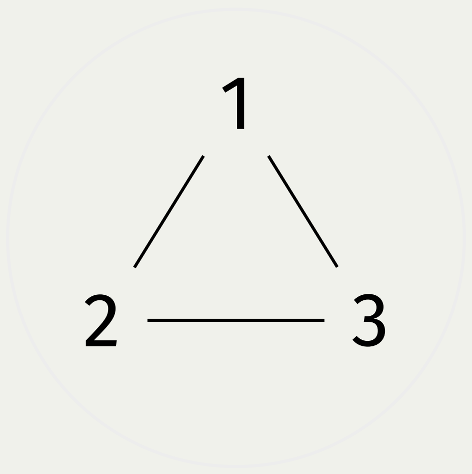
Linear preprocessing!

Hardness of subqueries

$Q_1 = R(1,2,3) \wedge S(1,2) \wedge T(2,3) \wedge U(3,1)$

$Q_2 = S(1,2) \wedge T(2,3) \wedge U(3,1)$





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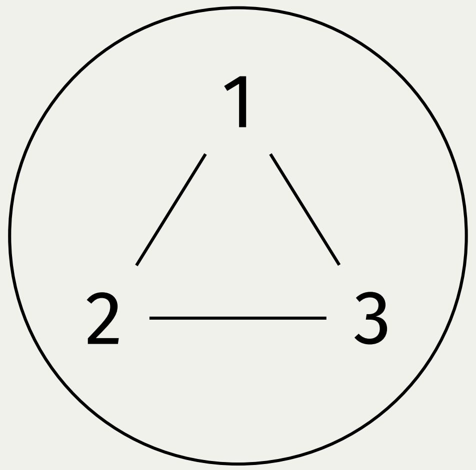
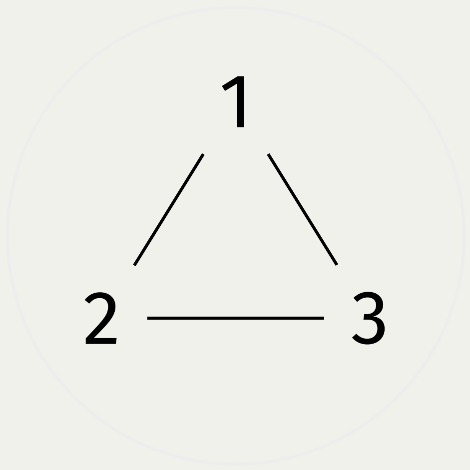
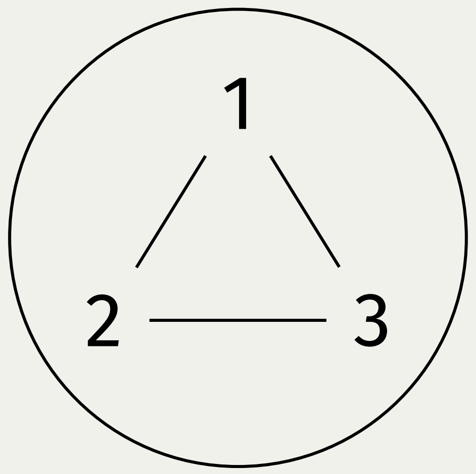




linear preprocessing

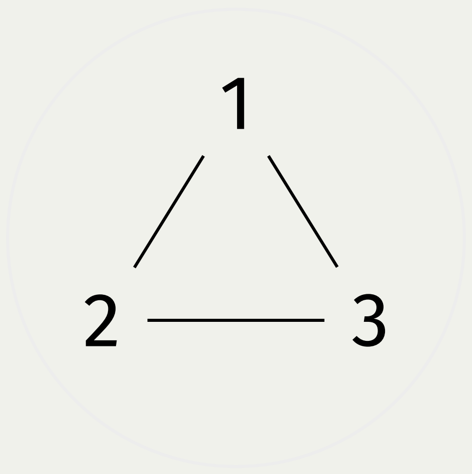
Hardness of subqueries

$Q_1 = R(1,2,3) \wedge S(1,2) \wedge T(2,3) \wedge U(3,1)$



linear preprocessing

$Q_2 = S(1,2) \wedge T(2,3) \wedge U(3,1)$



non-linear preprocessing

Hardness of subqueries

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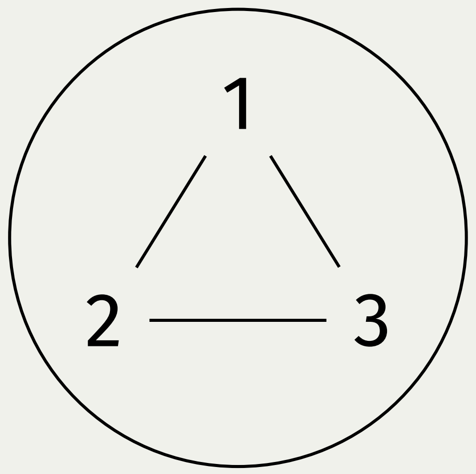
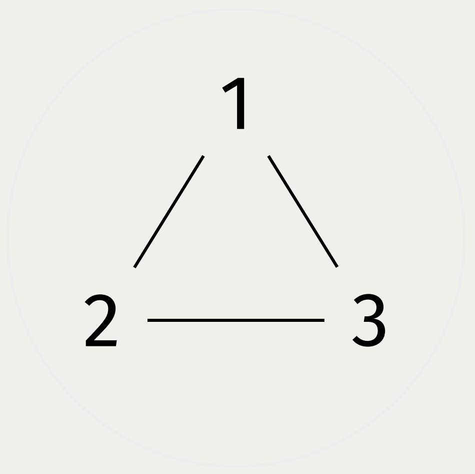




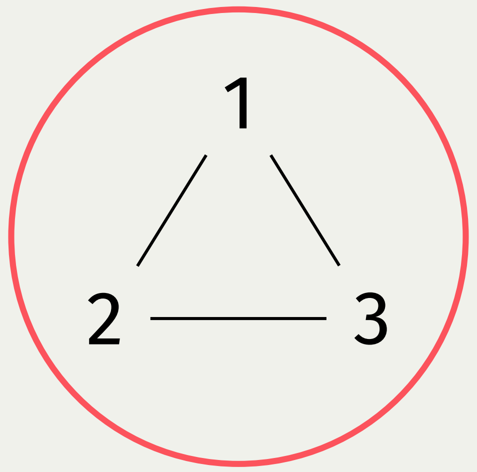
linear preprocessing

non-linear preprocessing

Subqueries may be harder to solve than the query itself!

Subqueries and negative atoms

$Q_1{}' = \neg R(1,2,3)$
$\wedge S(1,2) \wedge T(2,3) \wedge U(3,1)$



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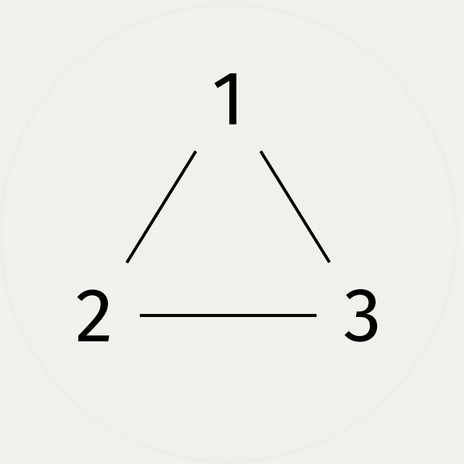

non-linear preprocessing (triangle)

Subqueries and negative atoms

$$Q_1{}' = \neg R(1,2,3) \wedge S(1,2) \wedge T(2,3) \wedge U(3,1)$$

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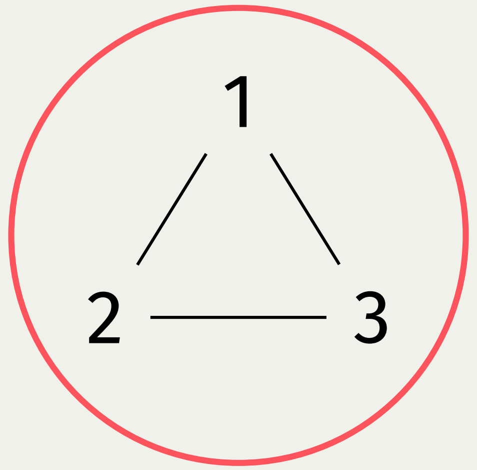
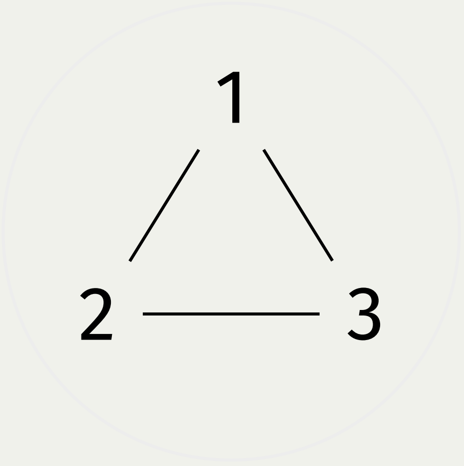


Equivalent to Q_2 if $R = \emptyset$



non-linear preprocessing (triangle)

Subqueries and negative atoms

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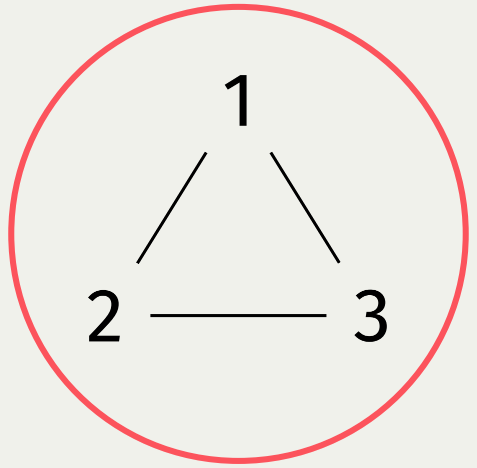
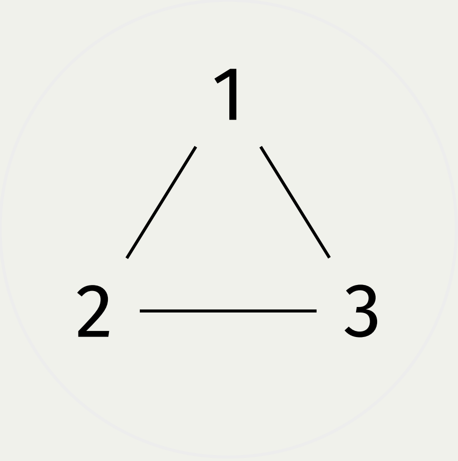




Equivalent to Q_2 if $R = \emptyset$

non-linear preprocessing (triangle)

DA for $Q = P \wedge N$ implies DA for $Q = P \wedge N'$ for every $N' \subseteq N$!

Measuring hardness of SJQ

Good candidate for $Q = Q^+ \wedge Q^-$:

Signed-HyperOrder Width

$$sfhow\,(\,Q, \pi\,) \;=\; \max_{Q' \subseteq Q^-} \iota\,(\,Q^+ \wedge Q', \pi\,)$$

For Q a (positive) JQ, and π a variable ordering, we can solve DA with

- Preprocessing $\tilde{O}\left(\,|\mathbb{D}|^{\iota\,(\,Q, \pi\,)}\,\right)$
- Access time $O\,(\,\log|\mathbb{D}|\,)$

Measuring hardness of SJQ

Good candidate for $Q = Q^+ \wedge Q^-$:

Signed-HyperOrder Width

$$sfhow\,(\,Q, \pi\,) \;=\; \max_{Q' \subseteq Q^-} \iota\,(\,Q^+ \wedge Q', \pi\,)$$

For Q a **signed JQ** , and π a variable ordering, we can solve DA with

- Preprocessing $\tilde{O}\left(\,|\mathbb{D}|^{sfhow\,(\,Q, \pi\,)}\,\right)$
- Access time $O\,(\,\log|\mathbb{D}|\,)$

Our contribution : new island of tractability for Signed JQ!

A word on sfhow

Signed Fractional HyperOrder Width (and incidentally, our result) generalizes:

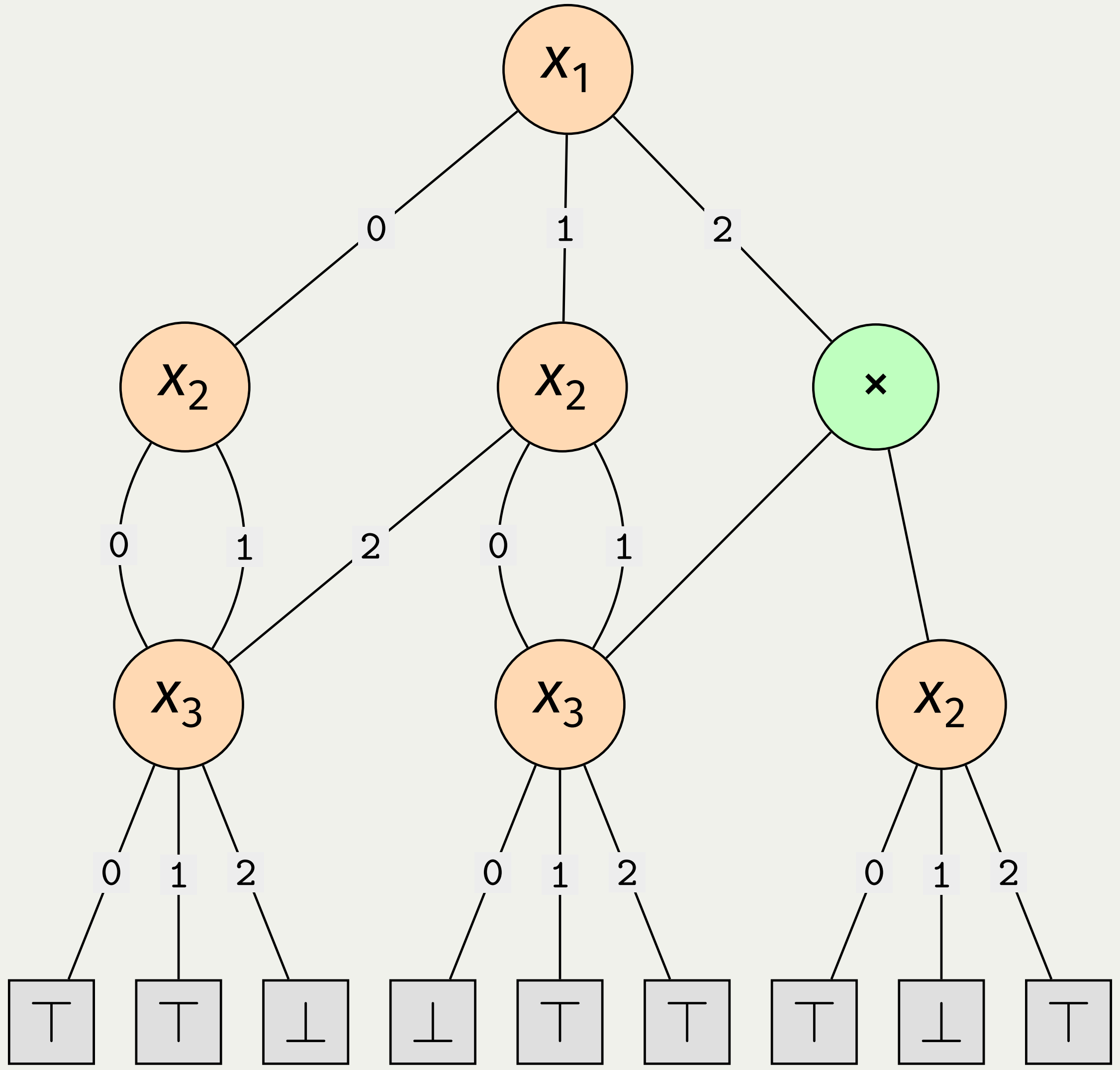
- β-acyclicity (#SAT and #NCQ are already known tractable)
- *signed*-acyclicity (Model Checking for SCQ known to be tractable)
- Nest set width (SAT / Model Checking for NCQ known to be tractable)
- A *non-fractional version* `show` can be defined (better combined complexity)

Basically, everything that is known to be tractable on SCQ/NCQ.

1. *Understanding model counting for β-acyclic CNF-formulas*, J. Brault-Baron, F. C., S. Mengel
2. *De la pertinence de l'énumération: complexité en logiques propositionnelle et du premier ordre*, J. Brault-Baron
3. *Tractability Beyond β-Acyclicity for Conjunctive Queries with Negation*, M. Lanzinger

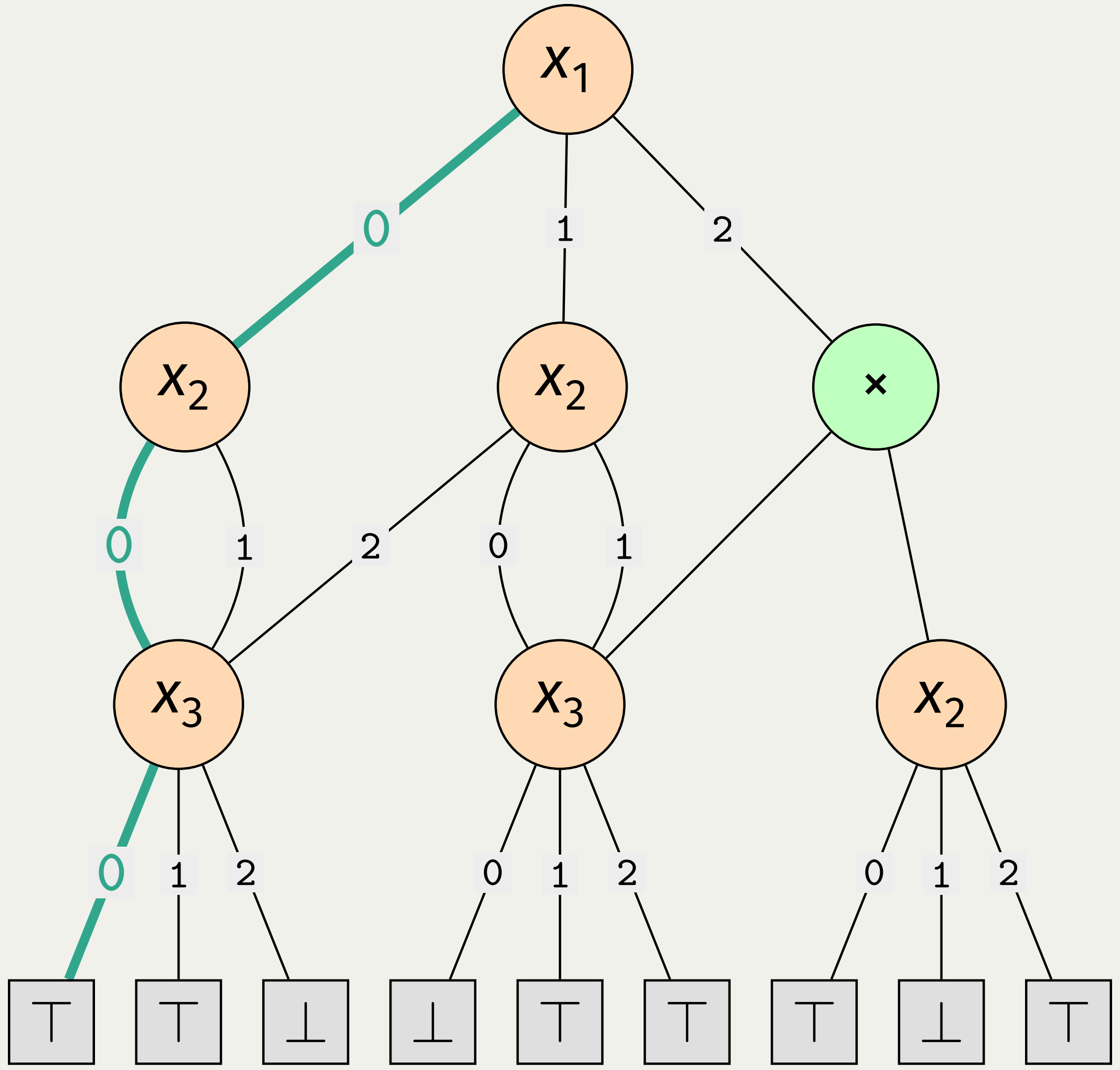
Our algorithm: a circuit approach

Relational Circuits



x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	1
1	0	2
1	1	1
1	1	2
1	2	0
1	2	1
2	0	1
2	0	2
2	2	1
2	2	2

Relational Circuits



x_1	x_2	x_3
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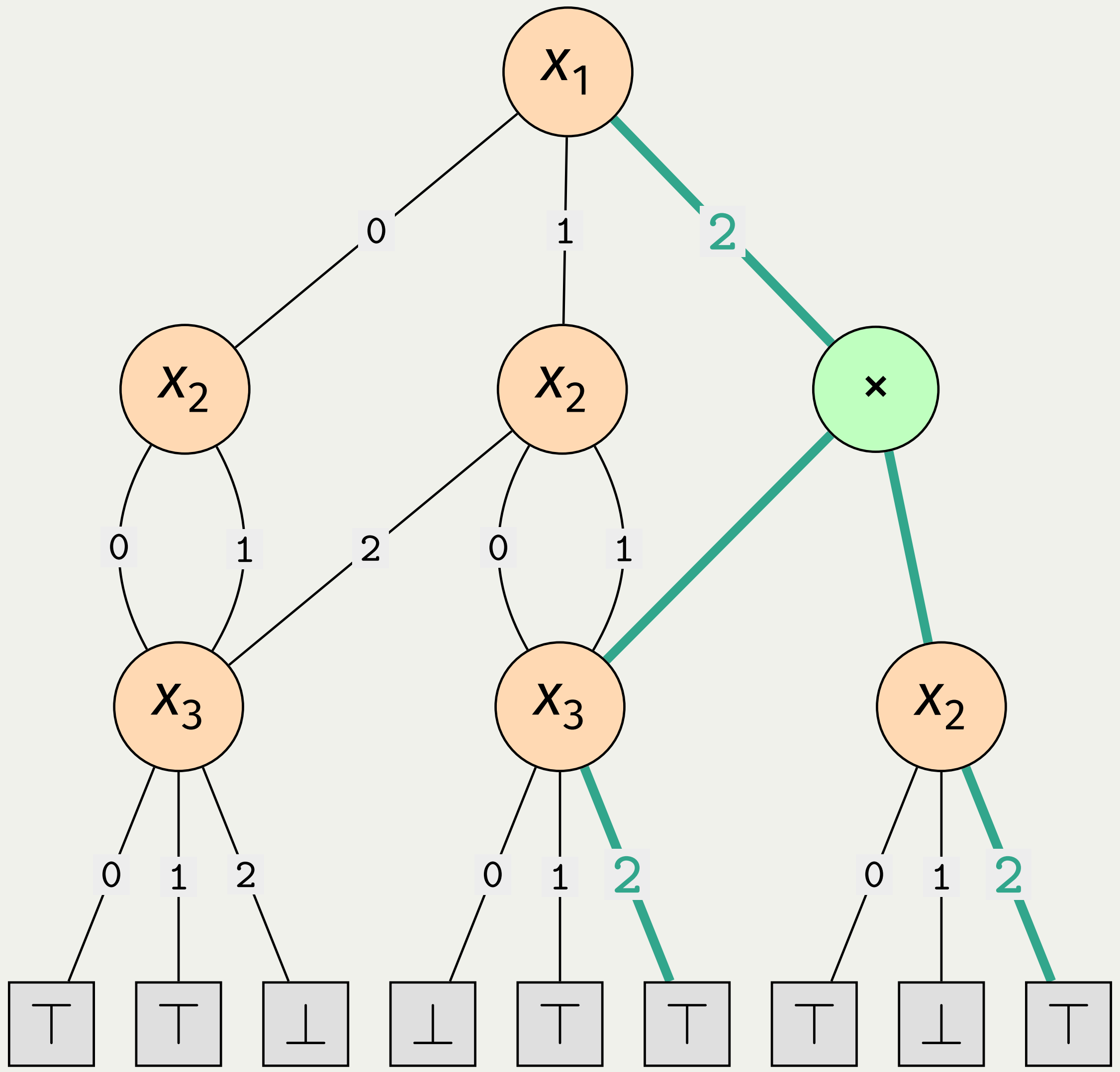
Relational Circuits



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2	0	2
2	2	1
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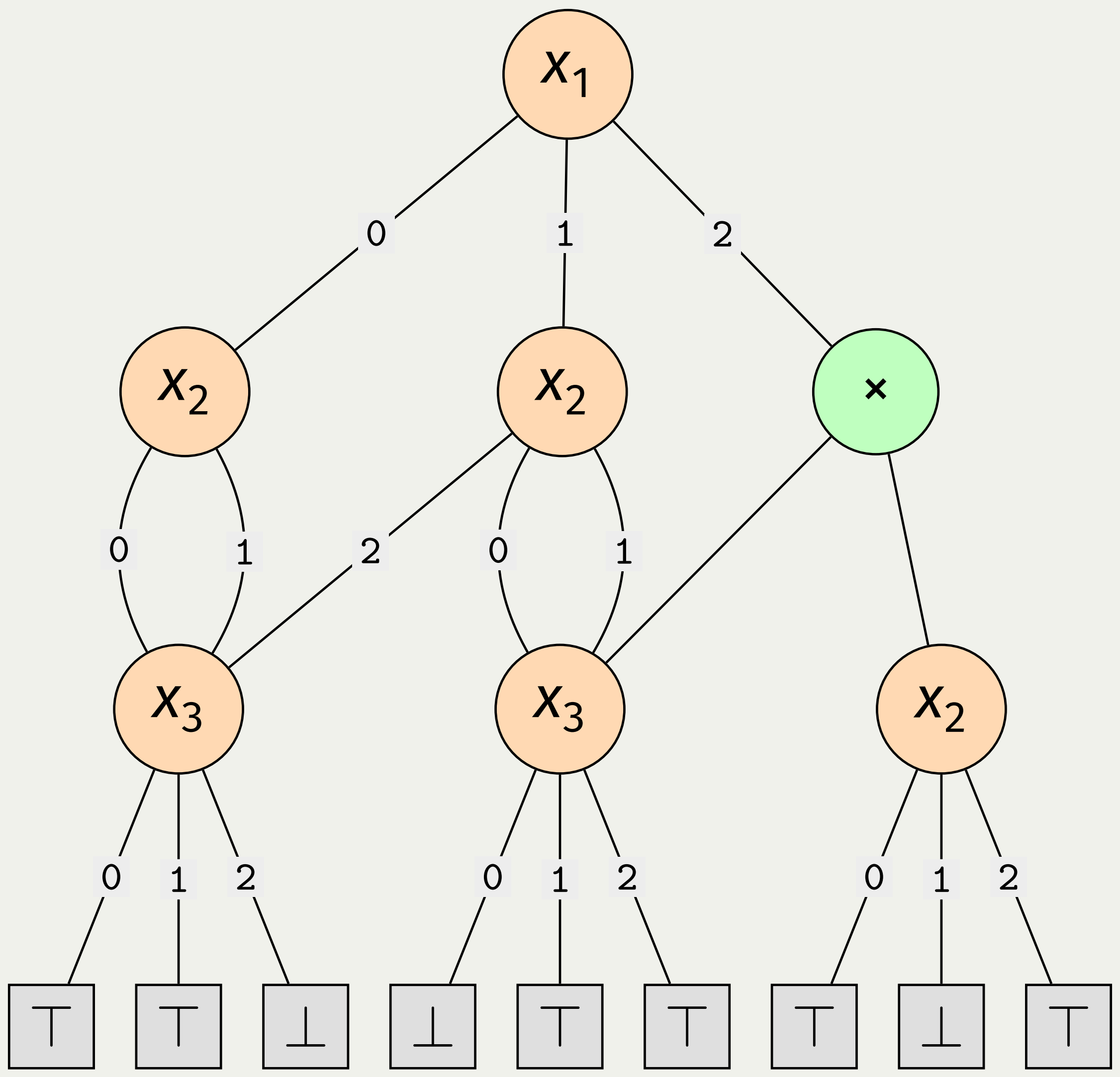
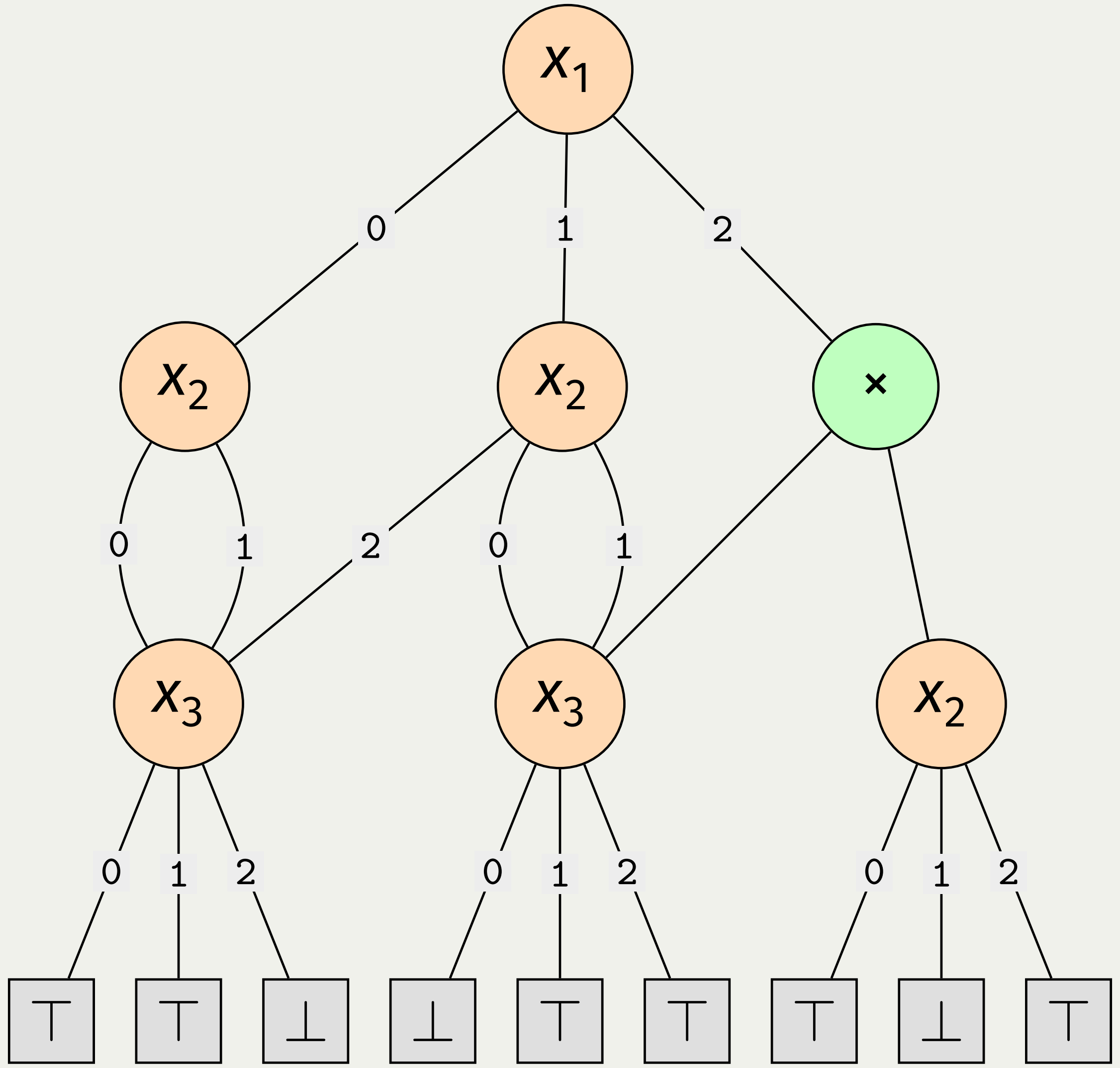
Ordered Relational Circuits



Factorized representation of relation $R \subseteq D^X$:

- **Inputs** gates : $\top$ & $\bot$
- **Decision** gates
- **Cartesian products**: $\times$ -gates

Ordered Relational Circuits

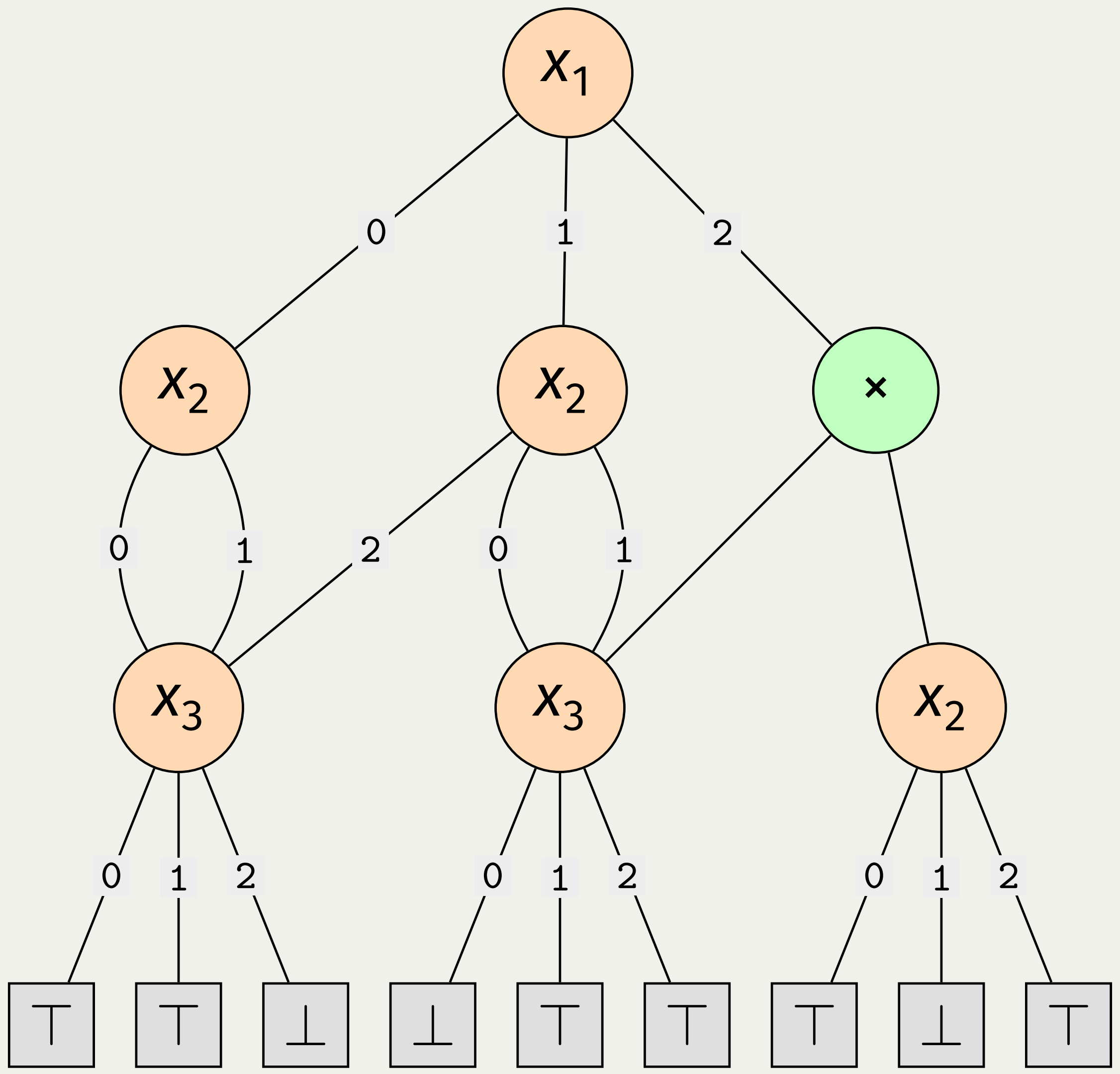


Factorized representation of relation $R \subseteq D^X$:

- **Inputs** gates : $\top$ & $\bot$
- **Decision** gates
- **Cartesian products**: $\times$ -gates

Ordered: decision gates below x_i only mention x_j with $j > i$.

Direct Access on Relational Circuits



For C on domain D, variables $x_1, \ldots, x_n$, DA possible :

- **Preprocessing**: $O\ (\ |C|\ \log\ |D|\ )$
- **Access time**: $O\ (\ n\ \log\ |D|\ )$

Preprocessing

Preprocessing

Idea : for each gate v over x_i and for each domain value d

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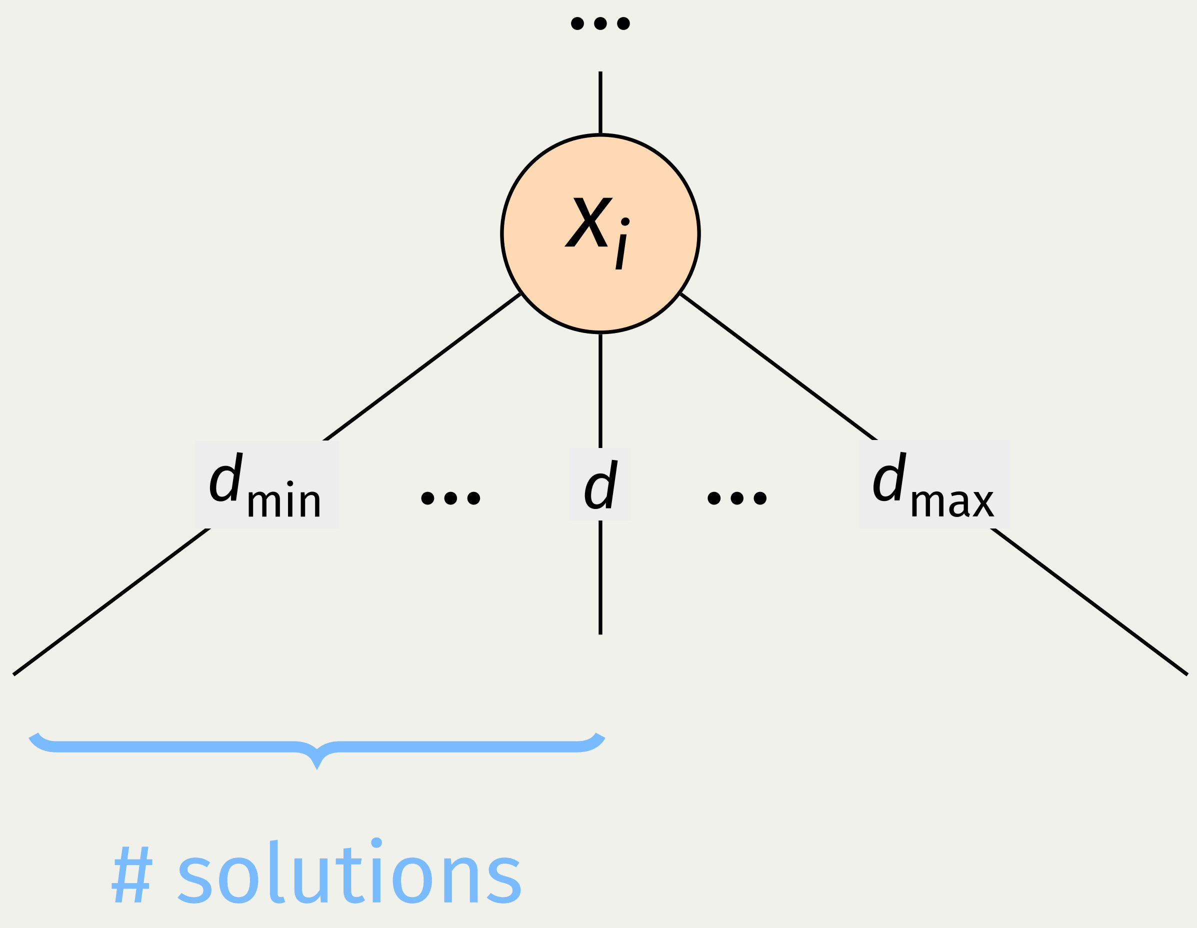
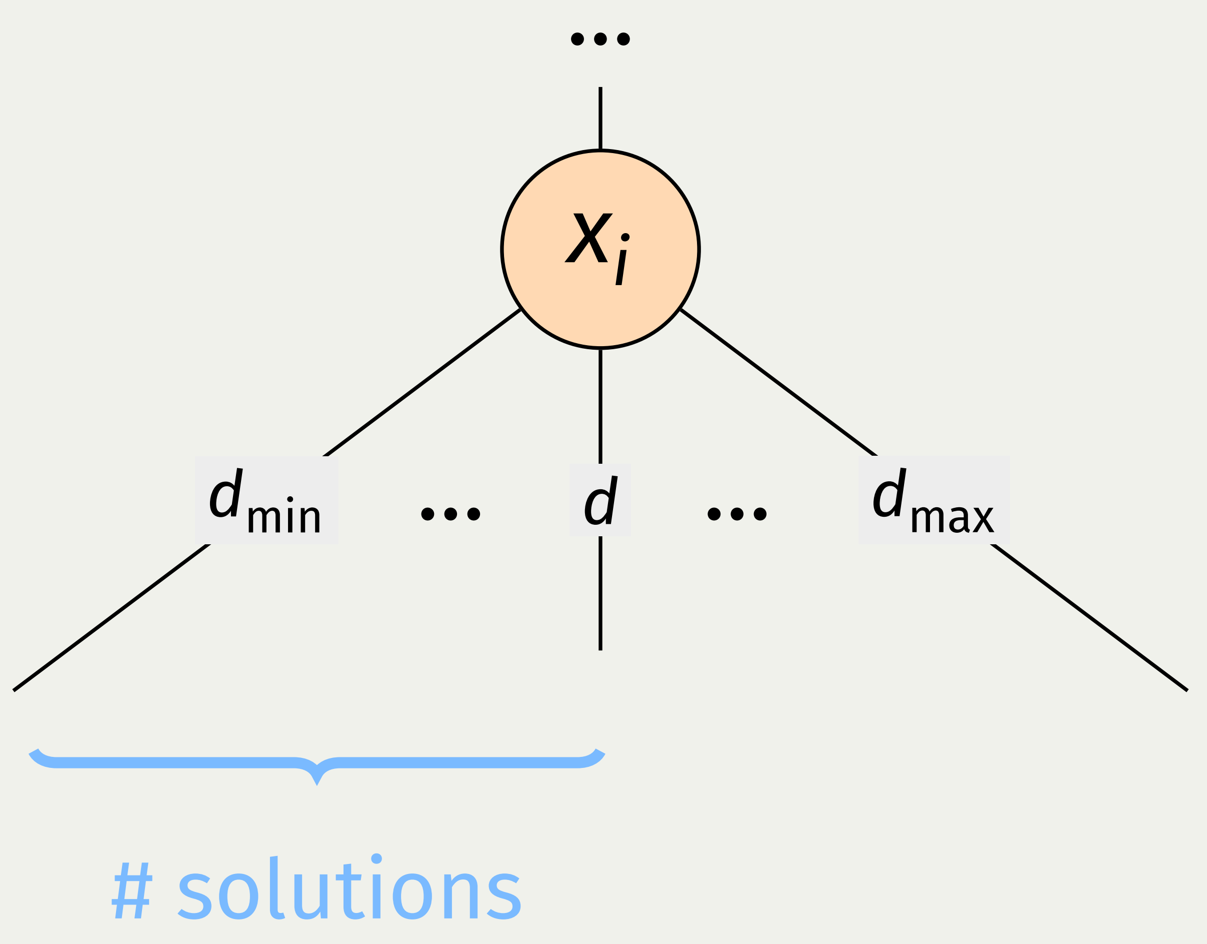


Preprocessing

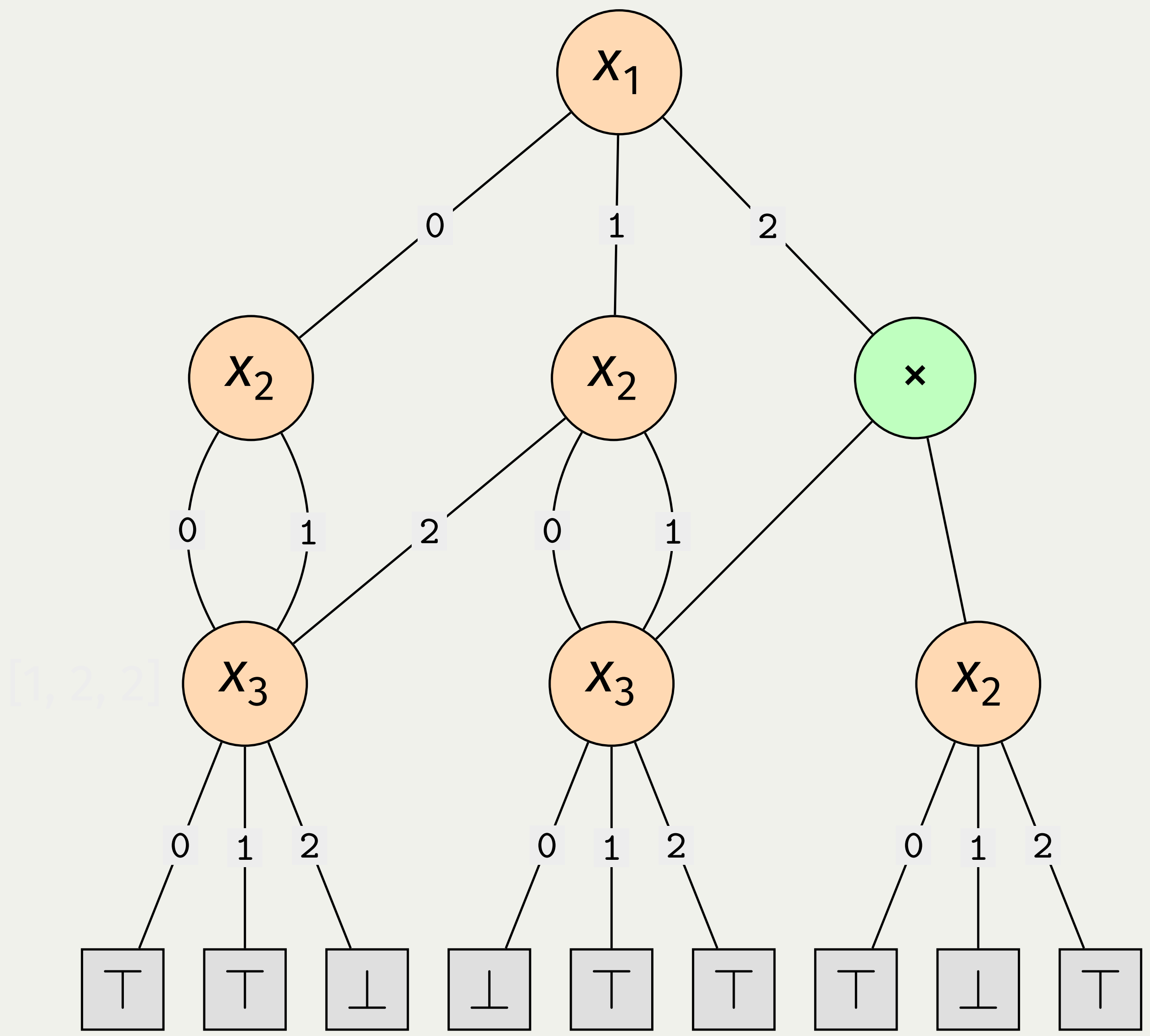
Idea : for each gate v over x_i and for each domain value d



compute the size of the relation where x_i is set to a value $d' \leq d$

Preprocessing

Preprocessing



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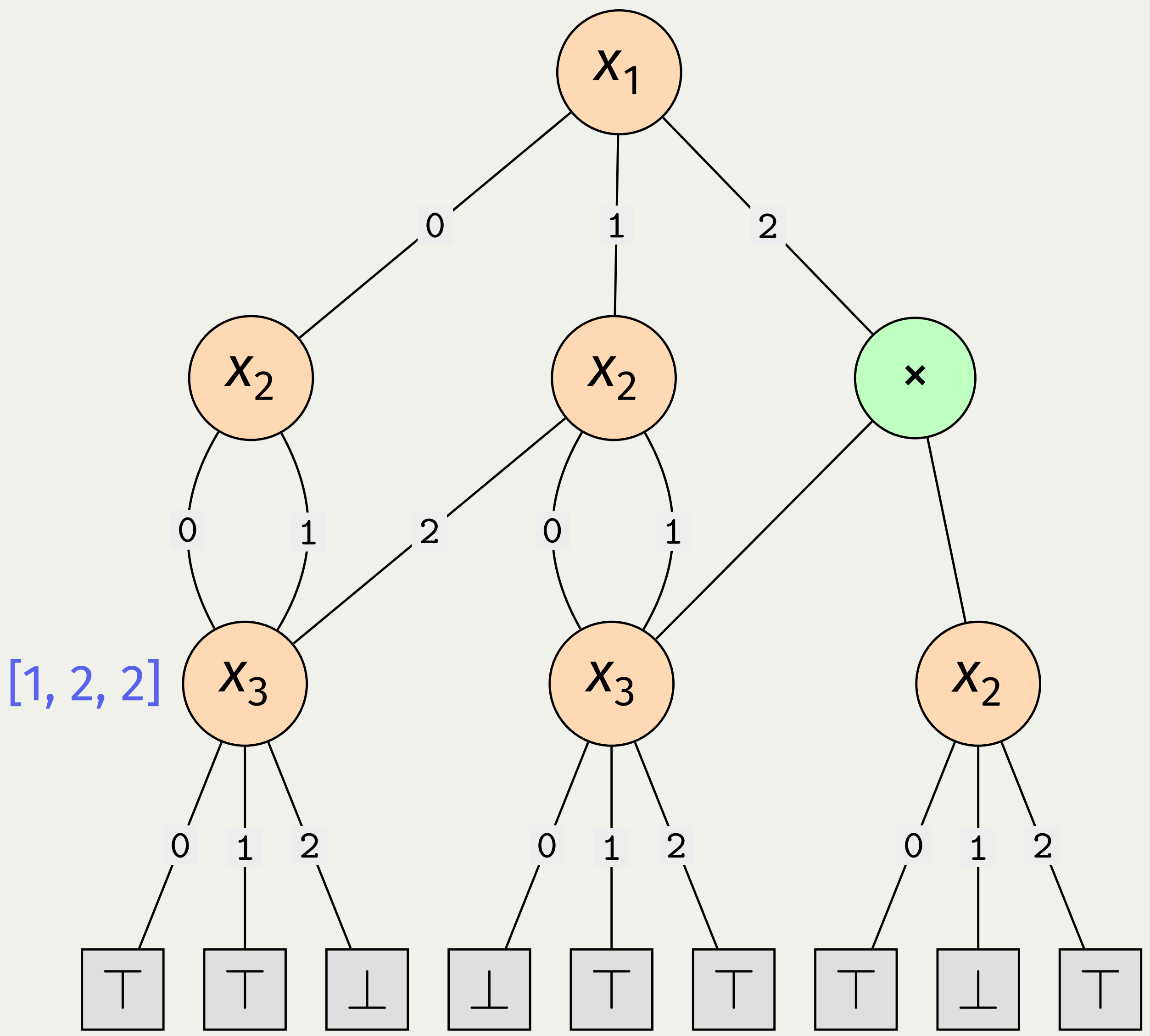
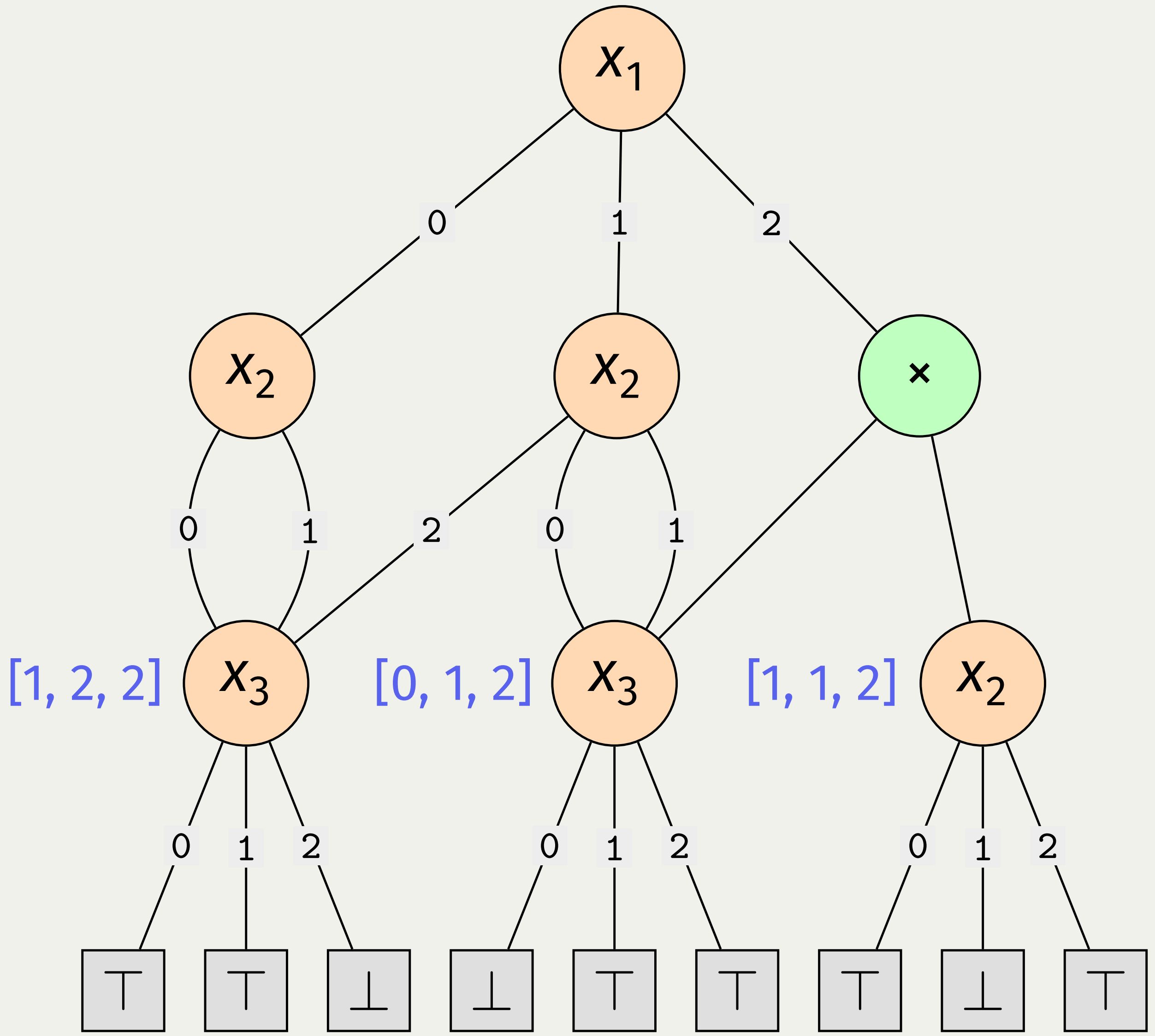
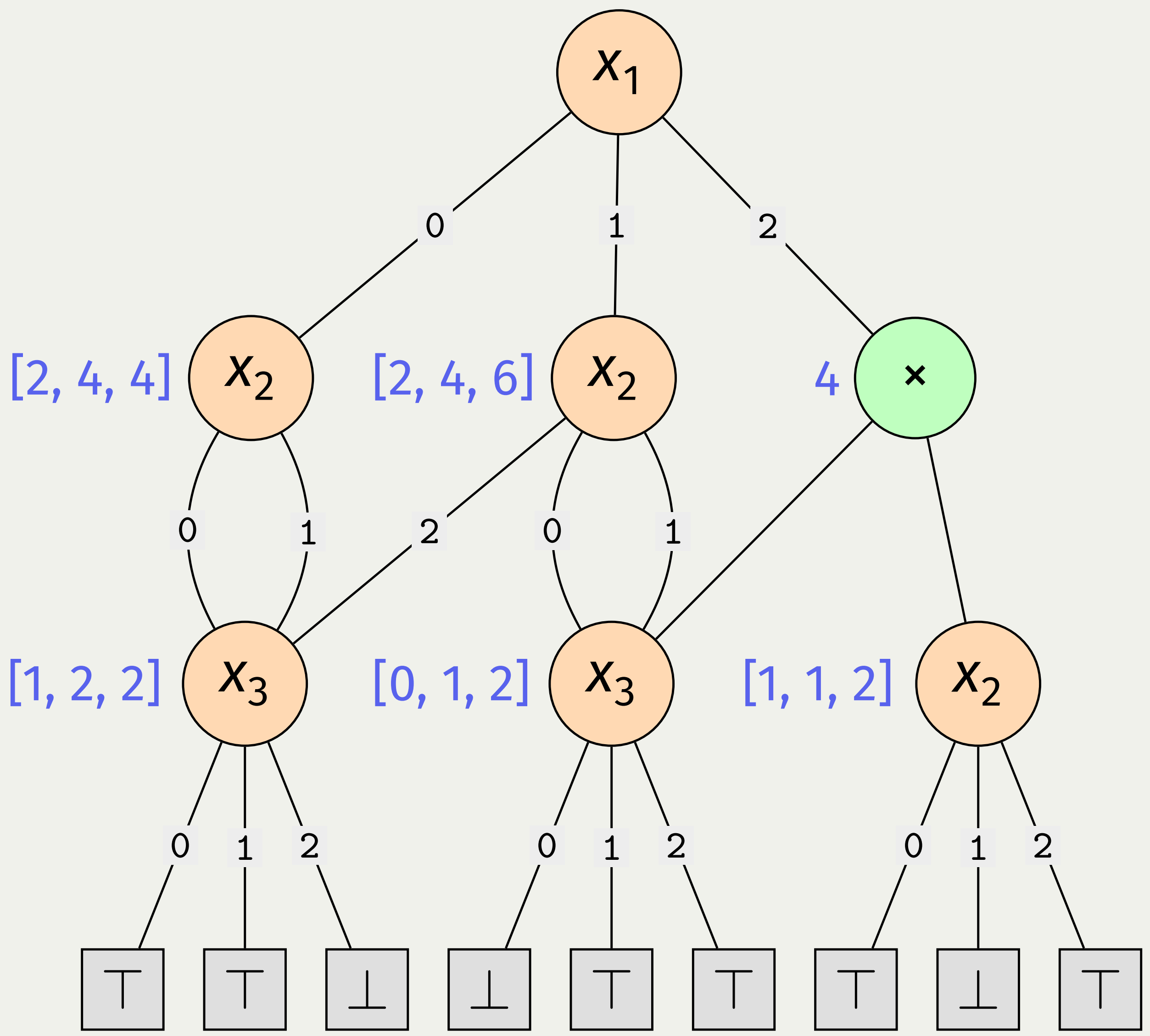


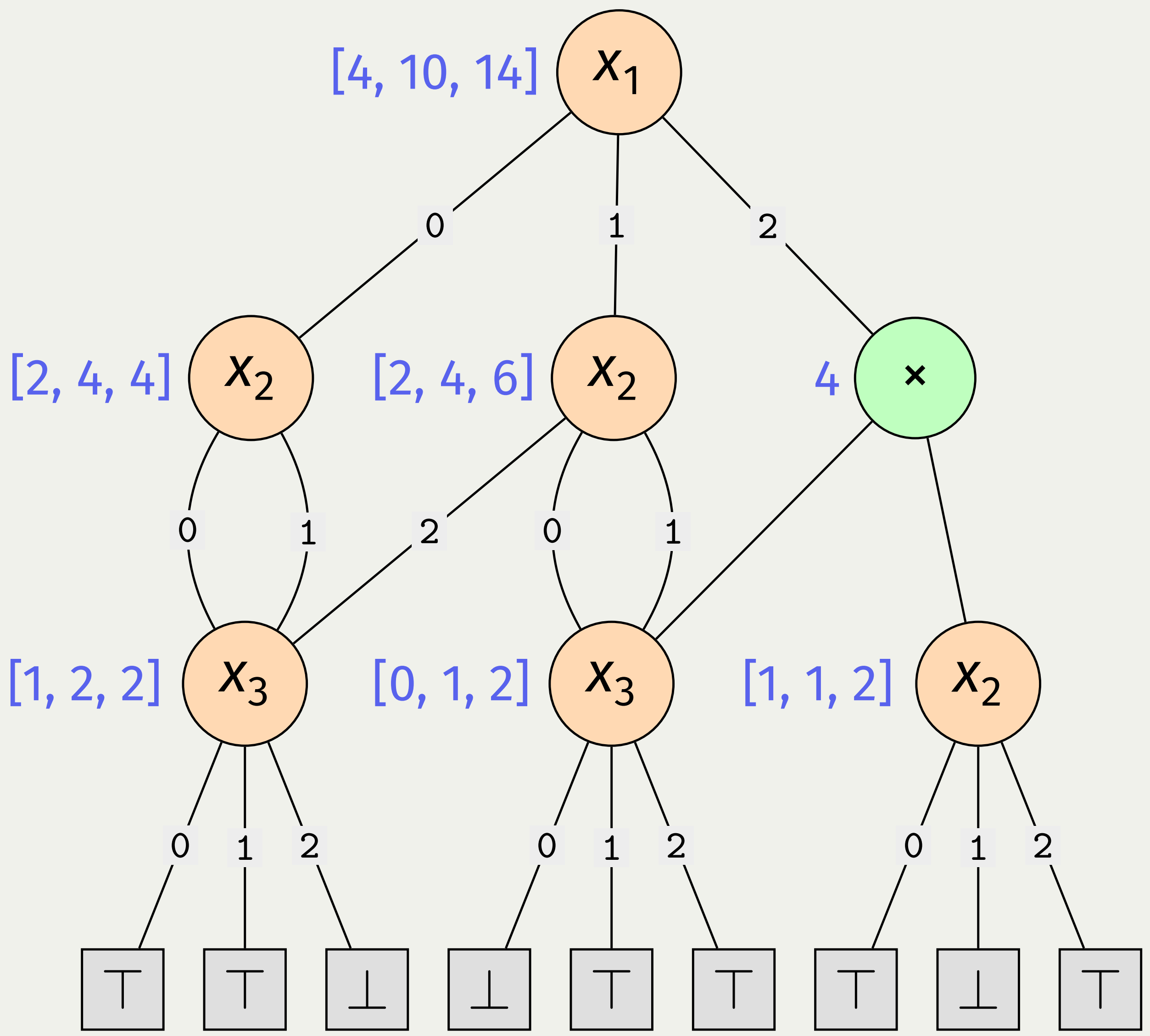
Preprocessing



Preprocessing



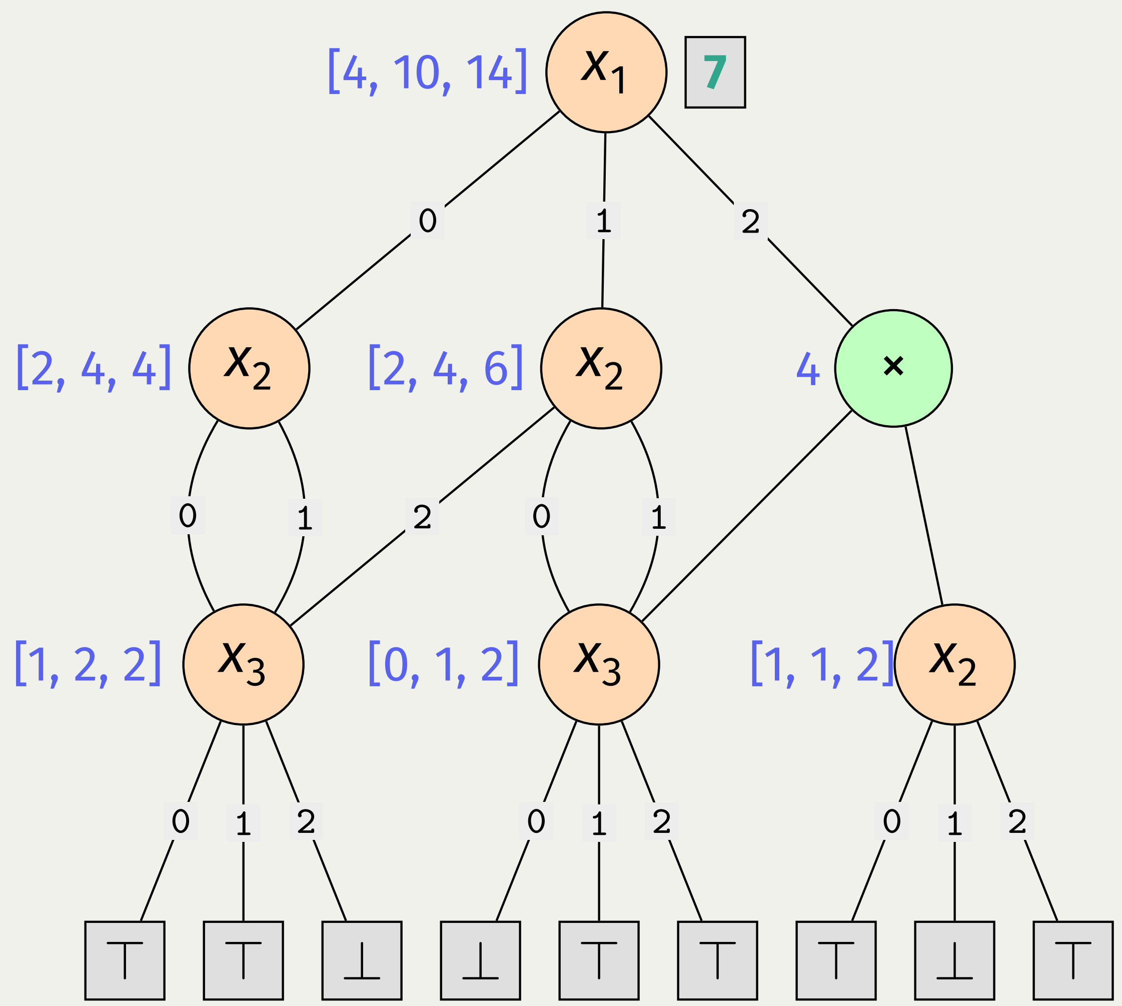
Preprocessing



Direct Access 7th solution

Compute the 7^{th} solution

Direct Access 7th solution



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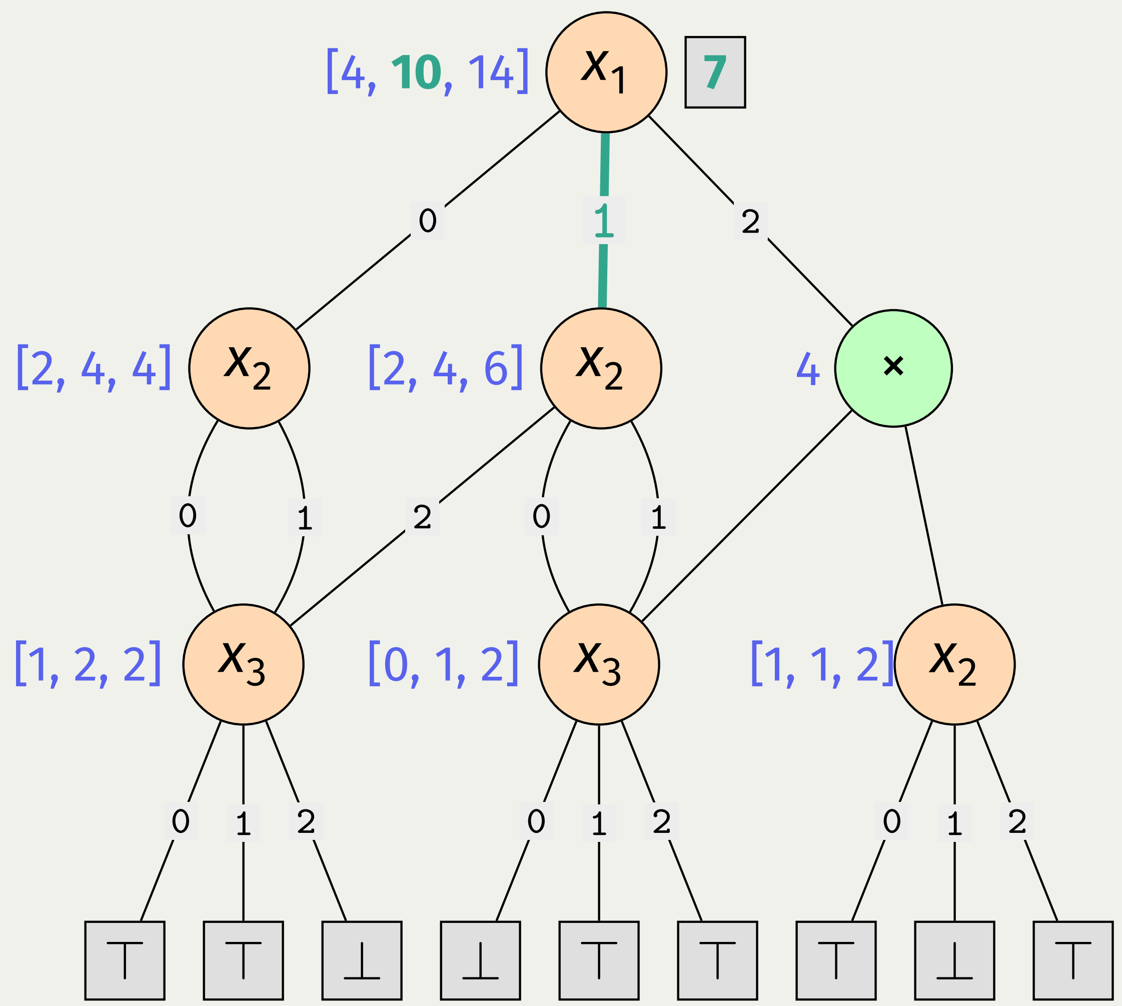

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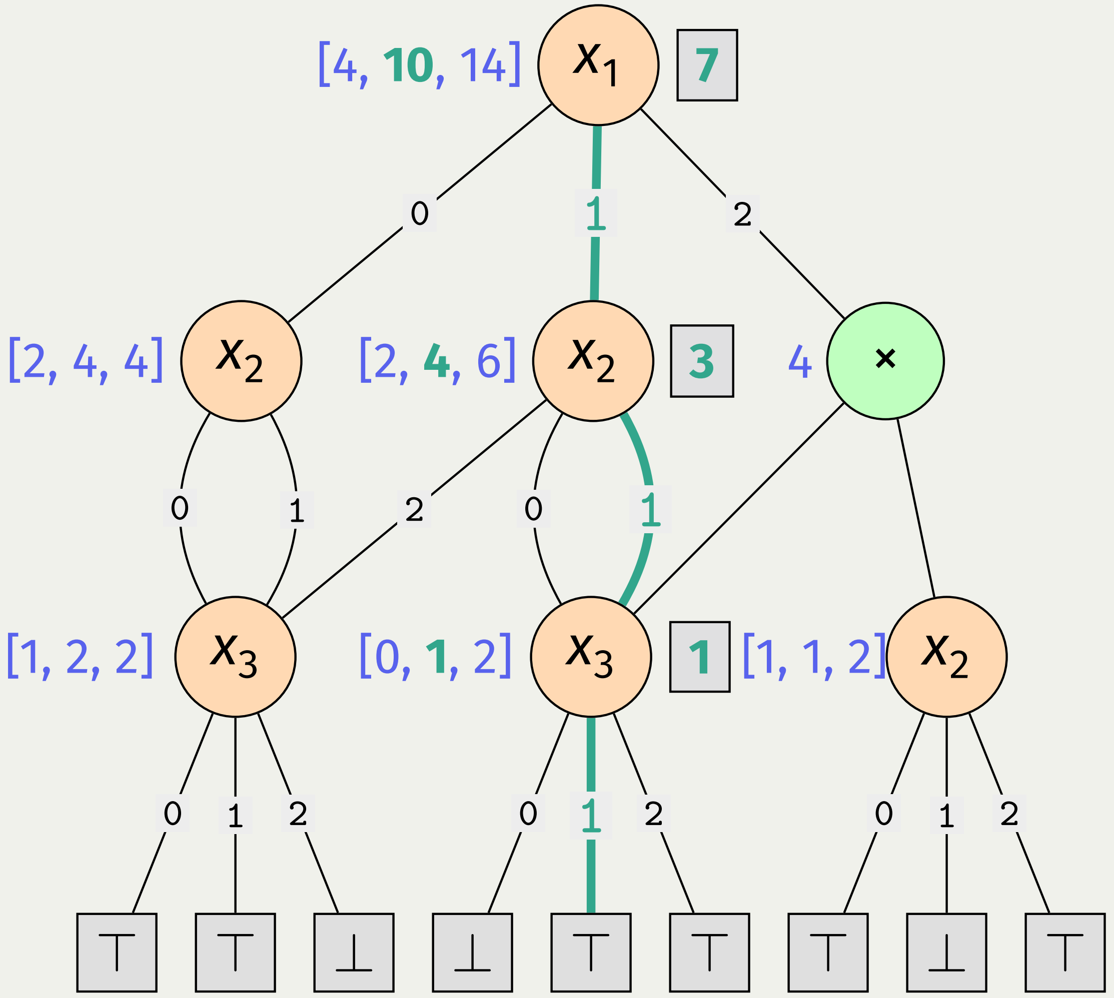
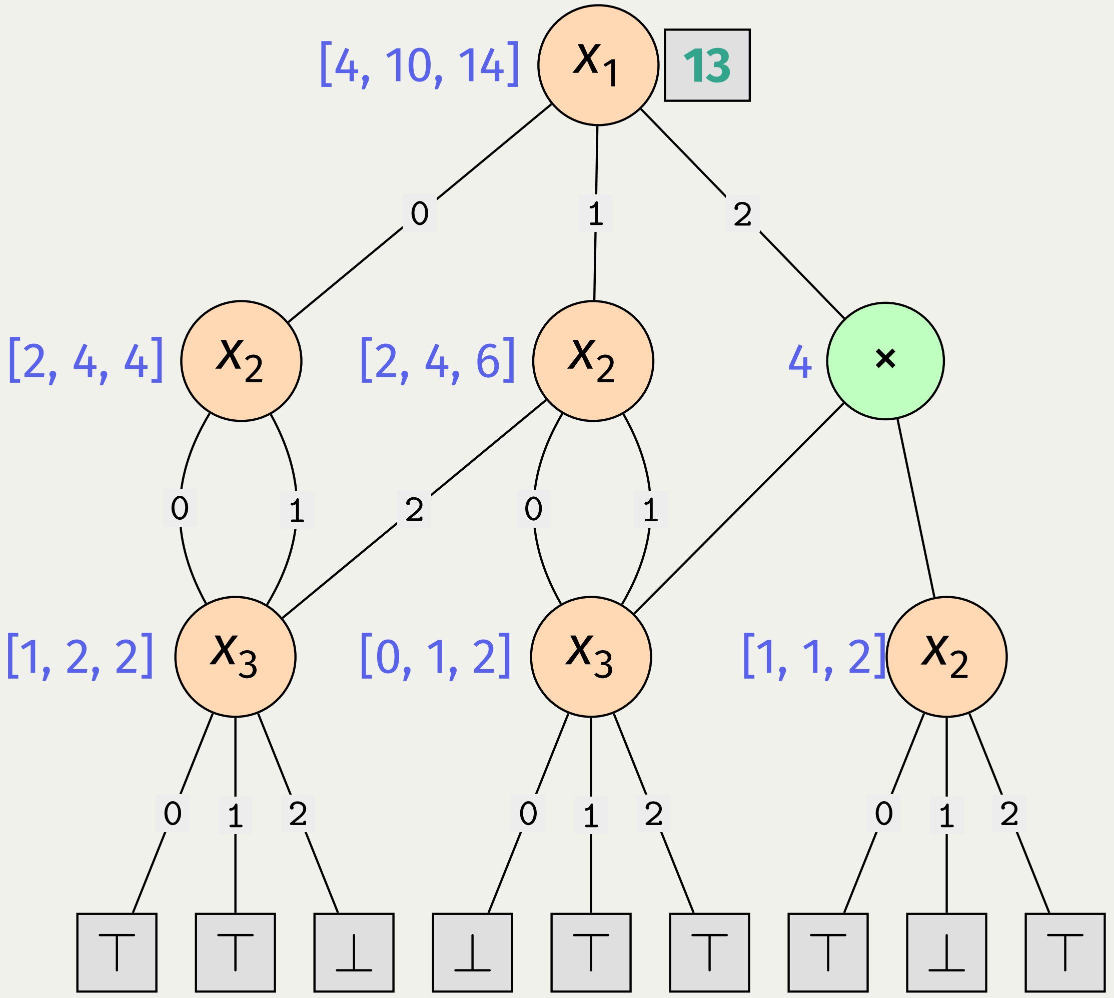


Compute the 7^{th} solution $\rightarrow$ 111

Direct Access the 13th solution

Compute the 13^{th} solution

Direct Access the 13th solution



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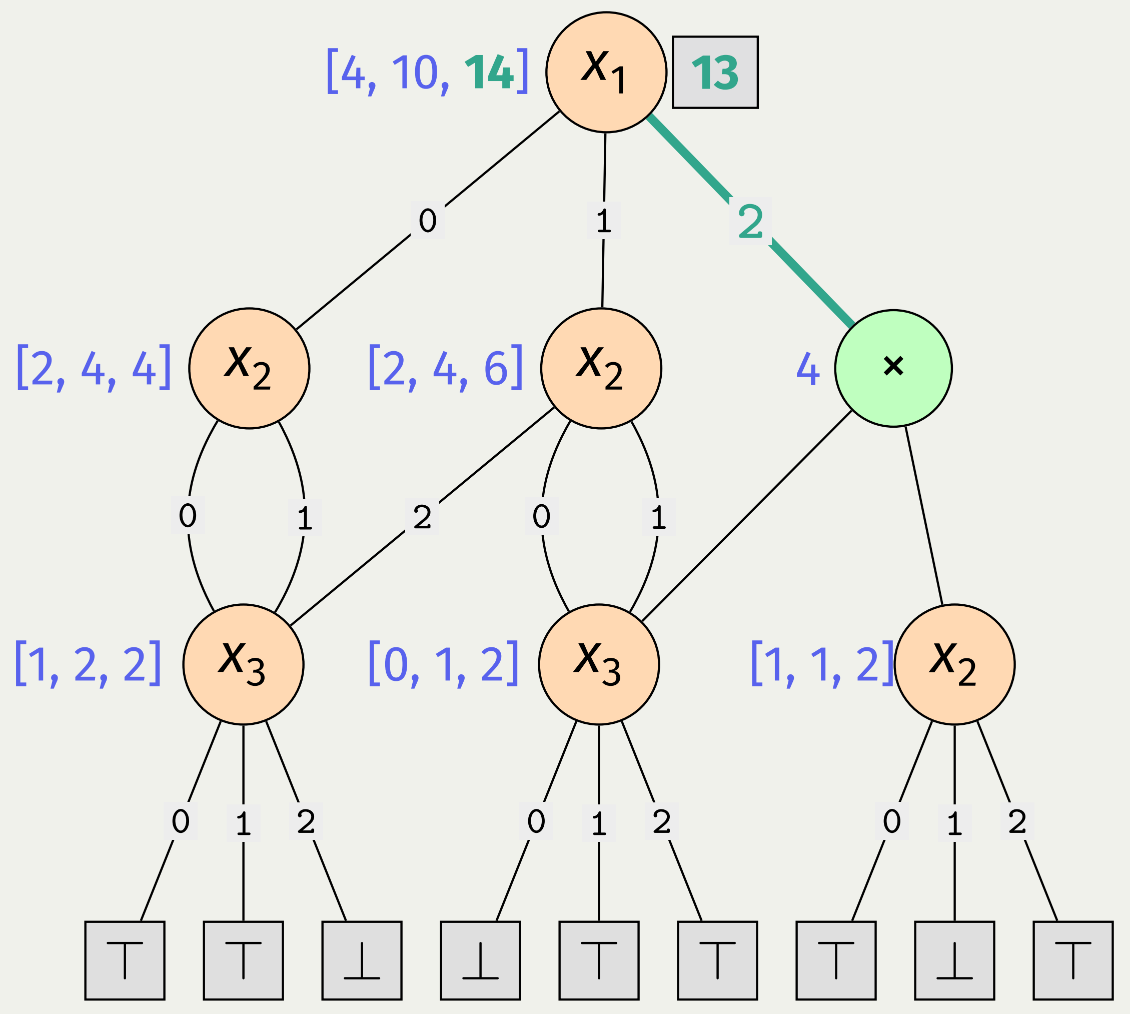

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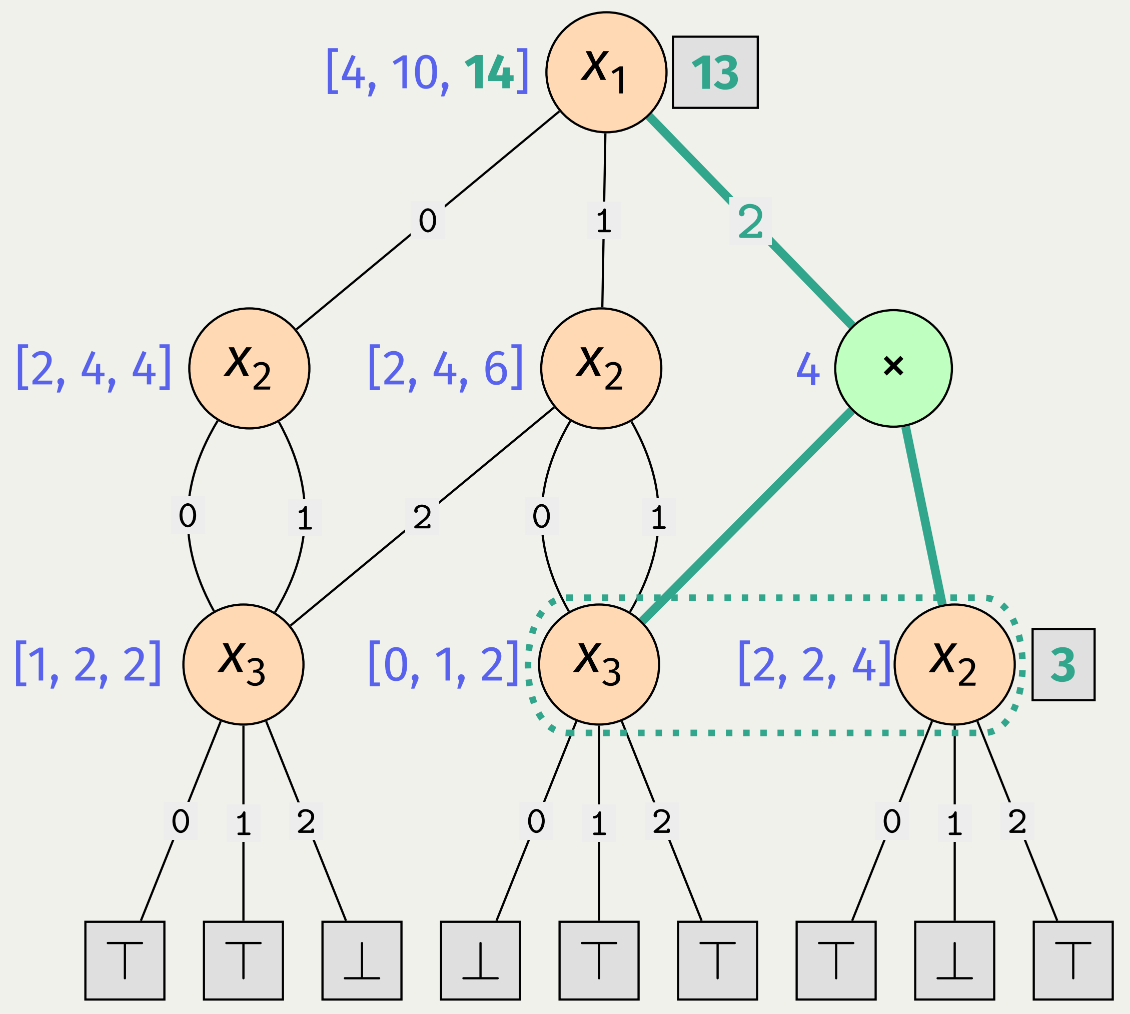

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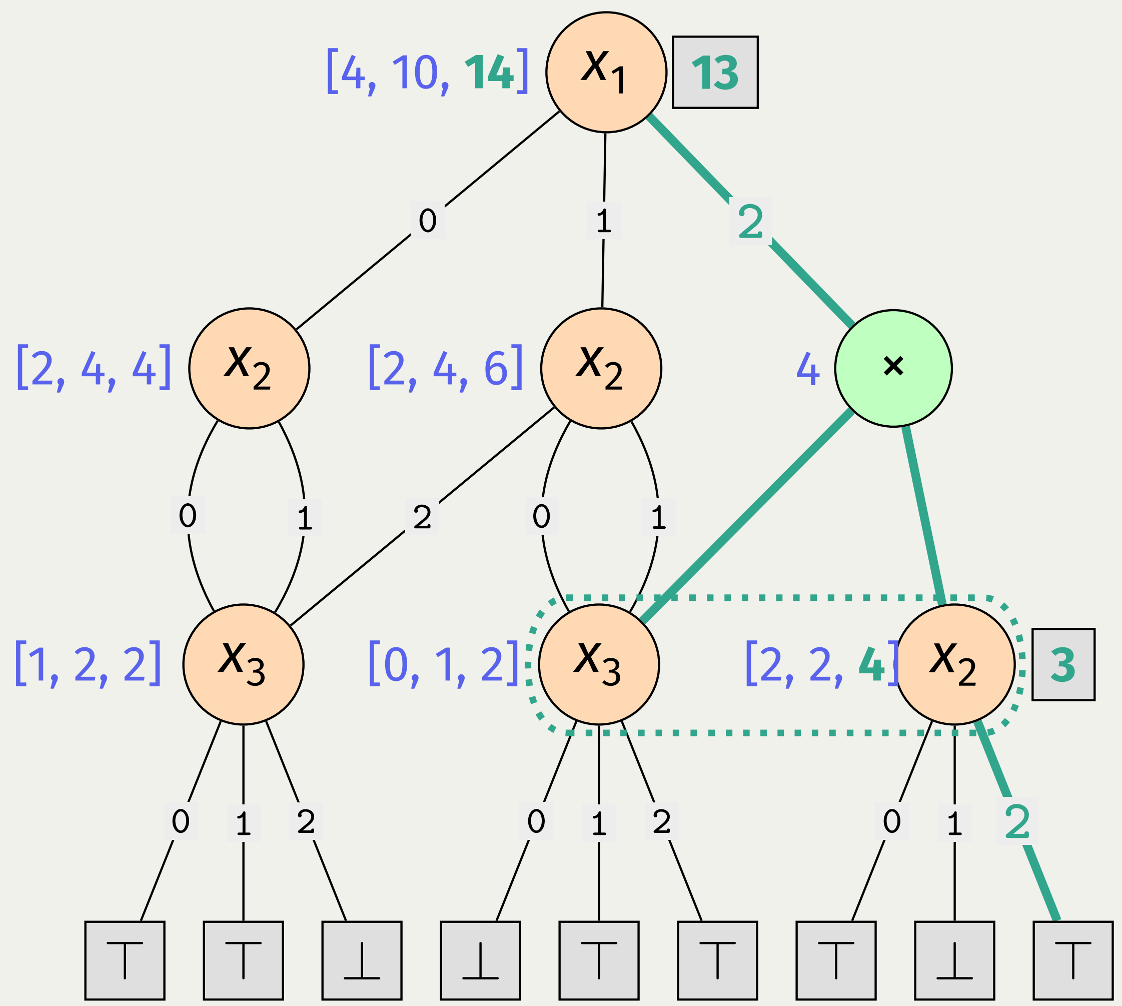

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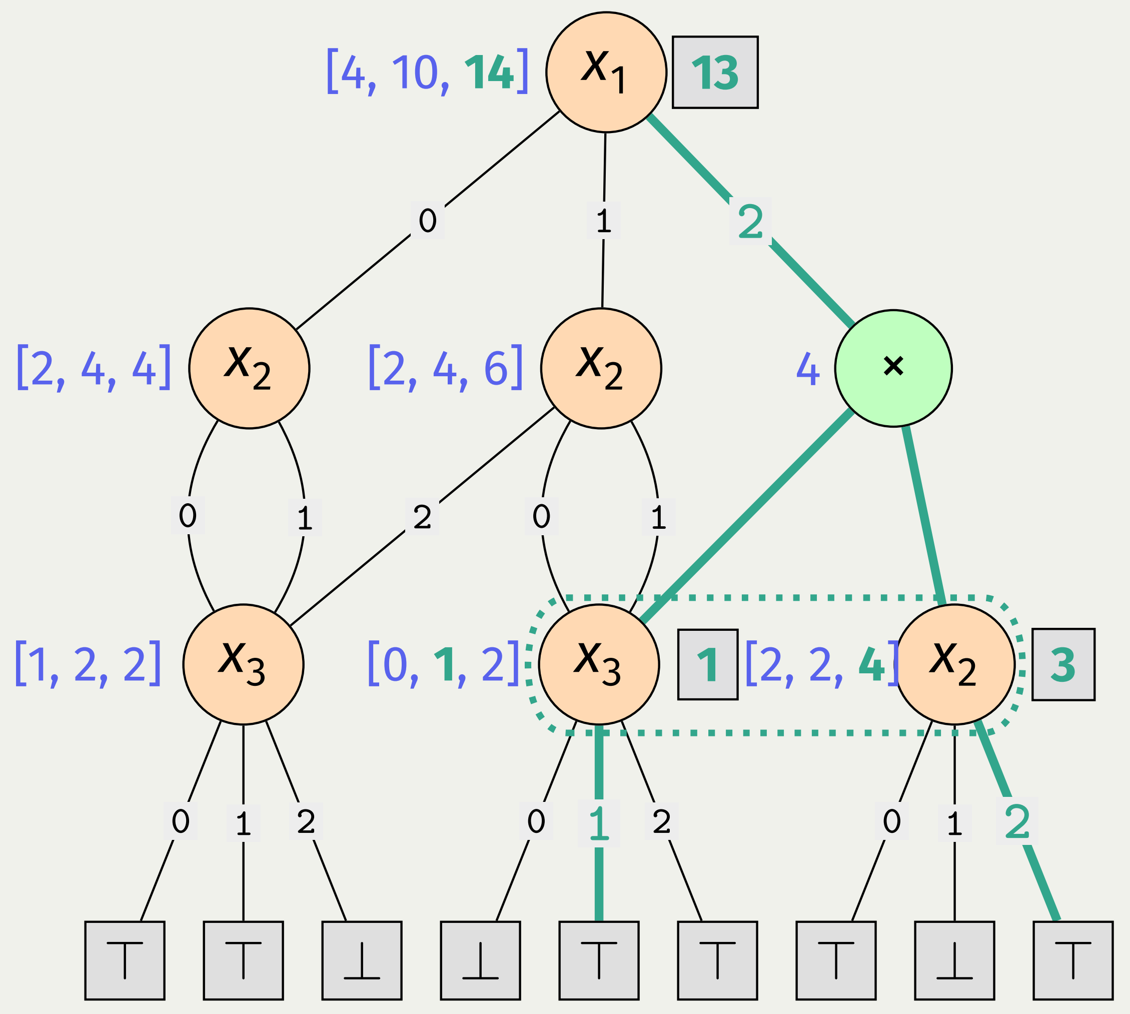

Compute the 13^{th} solution $\rightarrow$ 221

Solving DA for SCQ

SCQ $Q(x_1, \ldots, x_n)$, $\pi = (x_1, \ldots, x_n)$.

Preprocessing:

1. Construct π-ordered circuit C of size $\tilde{O}\left(|\mathbb{D}|^{sfhow(Q,\pi)} f(Q)\right)$
2. Preprocess C in time $O(|C| \log |\mathbb{D}|)$.

Direct Access :

1. Directly on C
2. in time $O(n \log |D|)$!

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Q, n considered constant here!

- **The hidden constants $f(Q)$ are exponential in $|Q|$ for bounded $sfhow(Q)$.**
- **But polynomial in Q for bounded $show(Q)$ (non fractional question).**

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Direct Access : $O(\log |\mathbb{D}|)$

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DPLL: building circuits

Compilation based on a variation of DPLL :

1. $Q(\mathbb{D}) = \biguplus_{d \in D} [x_1 = d] \times Q[x_1 = d](\mathbb{D})$
2. $Q(\mathbb{D}) = Q_1(\mathbb{D}) \times Q_2(\mathbb{D})$ if $Q = Q_1 \wedge Q_2$ with $var\left(Q_1\right) \cap var\left(Q_2\right) = \emptyset$
3. Top down induction + caching

https://florent.capelli.me/cytoscape/dpll.html

A comment on the complexity of DPLL

- If implemented this way, gives a $|\mathbb{D}|^{sfhow(Q)+1}$ complexity…
- Workaround: reencode the domain in binary and build a circuit iteratively testing **the bits of each variable**.

Going further

Related results

1. Extension to ∃SJQ:
    - Last variable in C can be **existentially projected** without increase in circuit size
    - Give DA for $\exists x_k, \ldots, x_n Q(x_1, \ldots, x_n)$.
2. Semi-ring Aggregation
    - $w \colon X \times D \to (\mathbb{K}, \oplus, \otimes)$
    - Compute $\bigoplus_{\tau \in Q(\mathbb{D})} \bigotimes_{x \in X} w(x, \tau(x))$
3. Lowerbounds: cannot do better than $|\mathbb{D}|^{sfhow(Q)}$ preprocessing.

Work in progress

Work in progress

1. Aggregation
   $Q(x_1, \ldots, x_k, F(x_{k+1}, \ldots, x_n))$ **, generalizing work by I. Eldar, N. Carmeli, B. Kimelfeld.**

Work in progress

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   $Q(x_1, ..., x_k, F(x_{k+1}, ..., x_n))$ **, generalizing work by I. Eldar, N. Carmeli, B. Kimelfeld.**
2. Understanding combined complexity for $sfhow(Q)$, the fractional version of $show$

Work in progress

1. Aggregation
   $Q(x_1, \ldots, x_k, F(x_{k+1}, \ldots, x_n))$ **, generalizing work by I. Eldar, N. Carmeli, B. Kimelfeld.**
2. Understanding combined complexity for $sfhow(Q)$, the fractional version of $show$
3. Comparing $show$ and β-hypertree width (the most general parameter for which the complexity is still unknown).